Bulk Viscosity and Contact Correlations in Attractive Fermi Gases

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The bulk viscosity determines dissipation during hydrodynamic expansion. It vanishes in scale invariant fluids, while a nonzero value quantifies the deviation from scale invariance. For the dilute Fermi gas the bulk viscosity is given exactly by the correlation function of the contact density of local pairs. As a consequence, scale invariance is broken purely by pair fluctuations. These fluctuations give rise also to logarithmic terms in the bulk viscosity of the high-temperature nondegenerate gas. For the quantum degenerate regime I report numerical Luttinger-Ward results for the contact correlator and the dynamical bulk viscosity throughout the BEC-BCS crossover. The ratio of bulk to shear viscosity ζ/η is found to exceed the kinetic theory prediction in the quantum degenerate regime. Near the superfluid phase transition the bulk viscosity is enhanced by critical fluctuations and has observable effects on dissipative heating, expansion dynamics, and sound attenuation.

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The bulk viscosity is a fundamental transport property which determines friction and dissipation in fluids during hydrodynamic expansion [1,2]. In particular, scale invariant fluids can expand isotropically without dissipation and therefore have zero bulk viscosity [3]. In a generic interacting fluid, instead, a nonzero value of the bulk viscosity quantifies the breaking of scale invariance in physical systems ranging from QCD [4–7] to condensed matter [8–14]. An intriguing example is the two-dimensional dilute Fermi gas, where the classical model is scale invariant but a quantum scale anomaly breaks this symmetry [15–18]; this has recently been observed via breathing dynamics in cold-atom experiments [19–21].

The bulk viscosity is necessary to understand and predict the real-time evolution and hydrodynamic modes of dissipative quantum fluids and to quantitatively interpret current experiments. However, measurements of the bulk viscosity remain challenging even for classical fluids [22]. Now a novel experimental probe via the dissipative heating rate due to a change in scattering length has been proposed for atomic gases [14]. It is therefore important to compute the bulk viscosity theoretically for quantum gases, which moreover includes predictions for the classical gas in the high-temperature limit.

The bulk viscosity is defined as the correlation function of local pressure (the trace of the stress tensor). Since it vanishes in a scale invariant system, only the scale breaking part of the pressure contributes, the so-called trace anomaly [14,23,24]. This provides a formal link between the breaking of scale invariance and bulk viscosity. The bulk viscosity of the nonrelativistic, strongly interacting Fermi gas has been calculated from kinetic theory in the nondegenerate high-temperature limit [11,12] and in the low-temperature superfluid state [25,26]. Its value is largest in the strongly coupled region of the BEC-BCS crossover [27] near unitarity, but not precisely at unitarity where it must vanish by scale invariance [3,9,28]. Furthermore, hydrodynamic fluctuations give rise to nonanalytic corrections to the bulk viscosity at small frequencies [23,29]. However, key open questions include the bulk viscosity in degenerate Fermi gases at strong interaction, the relative importance of bulk and shear viscosity, and critical scaling near the superfluid phase transition.

In this work, I rewrite the bulk viscosity of the dilute Fermi gas as a correlation function of the contact density of local fermion pairs. This exact mapping explicitly links the bulk viscosity to pairing fluctuations as the relevant degrees of freedom and provides a genuine strong-coupling formulation which is valid in the whole BEC-BCS crossover including the quantum critical regime [30–32]. New results include (a) dominant logarithmic corrections to the bulk viscosity at high temperature, (b) numerical Luttinger-Ward results for the quantum degenerate gas throughout the BEC-BCS crossover predict a large bulk viscosity well observable with current experimental technology, (c) the transport ratio of bulk to shear viscosity deviates from the kinetic theory prediction in the quantum degenerate regime, and (d) critical scaling near the superfluid transition is less singular than predicted [29,33], but pairing fluctuations dynamically enhance the scale anomaly.

Bulk viscosity.—The bulk viscosity ζ is defined as the stress correlation function [8,34,35]

$$\zeta(\omega) = -\frac{1}{\omega d^2}\mathrm{Im}\int_0^\infty dt e^{i\omega t}\int d^d x\langle[\hat{\Pi}_{ii}(\boldsymbol{x},t),\hat{\Pi}_{jj}(0,0)]\rangle, \tag{1}$$

where the trace of the stress tensor $\hat{\Pi}_{ii}(\boldsymbol{x},t) = d\cdot\hat{P}$ determines the pressure operator $\hat{P}$ in dimension d. The two-component dilute Fermi gas is described by the Hamiltonian density [27]

$$\hat{\mathcal{H}} = \sum_\sigma \psi_\sigma^\dagger \left(-\frac{\nabla^2}{2m}\right)\psi_\sigma + g_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow. \quad (2)$$

The first term denotes the kinetic energy with fermion operators $\psi_\sigma(\boldsymbol{x}, t)$. The attractive contact interaction in the second term is characterized by the s-wave scattering length a. For a given value of a, the bare coupling strength g_0 is determined according to $g_0^{-1} = -(m/2\pi)\ln(a\Lambda)$ in two dimensions (2D) and $g_0^{-1} = (m/4\pi)(1/a - 2\Lambda/\pi)$ in 3D, with ultraviolet momentum cutoff Λ. The trace of the stress tensor is given by the scale variation of the Hamiltonian [17],

$$d \cdot \hat{P} = \hat{\Pi}_{ii} = [\hat{\mathcal{H}}, i\hat{D}] = 2\hat{\mathcal{H}} + \begin{cases} \frac{\hat{\mathcal{C}}}{2\pi m} & (2\mathrm{D}), \\ \frac{\hat{\mathcal{C}}}{4\pi m a} & (3\mathrm{D}), \end{cases} \quad (3)$$

where the dilatation operator $\hat{D} = \int d^d x \boldsymbol{x} \cdot m\boldsymbol{j}(\boldsymbol{x})$ generates scale transformations. The first term on the right-hand side is the scale invariant result $[\hat{\mathcal{H}}, i\hat{D}] = 2\hat{\mathcal{H}}$. If only this is present, the pressure is proportional to the Hamiltonian and commutes with itself in Eq. (1); hence the bulk viscosity $\zeta(\omega) \equiv 0$ vanishes identically in the scale invariant case [3,9,28,36].

The second term, in turn, is proportional to the local pair contact density $\hat{\mathcal{C}} = -m^2(\partial\hat{\mathcal{H}}/\partial g_0^{-1}) = m^2 g_0^2 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$ [37]. Scale invariance is recovered for the ideal quantum gas where $\hat{\mathcal{C}} = 0$, and also for the 3D unitary Fermi gas where $1/a = 0$ at the scattering resonance. A nonzero bulk viscosity therefore quantifies the breaking of scale invariance, which is generally expected in the interacting Fermi gas, except at unitarity.

Contact correlation.—By conservation of energy, the Hamiltonian in Eq. (3) does not contribute to the pressure commutator (1), and the bulk viscosity is given by the correlator of the scale breaking term. The scaling violation in the trace of the stress tensor is the so-called trace anomaly [14,23]

$$\hat{\Pi}_{\mathrm{an}} \equiv \hat{\Pi}_{ii} - 2\hat{\mathcal{H}} = c_d \hat{\mathcal{C}}, \quad (4)$$

where $c_d = -(\partial g_0^{-1}/\partial \ln|a|)/m^2$ denotes the scale variation of the bare coupling (beta function). For the dilute gas, the equilibrium bulk viscosity is thus exactly given by the contact correlator,

$$\zeta(\omega) = -\frac{c_d^2}{\omega d^2} \mathrm{Im} \int_0^\infty dt e^{i\omega t} \int d^d x \langle [\hat{\mathcal{C}}(\boldsymbol{x}, t), \hat{\mathcal{C}}(0,0)] \rangle. \quad (5)$$

The contact operator is the term in the Hamiltonian that couples to the scattering length. In linear response, the bulk viscosity thus captures how the local pair contact density at time t changes in response to a variation of the scattering length at earlier time $t = 0$ [14],

$$\chi(\boldsymbol{x}, t) \equiv \langle [\hat{\mathcal{C}}(\boldsymbol{x}, t), \hat{\mathcal{C}}(0,0)] \rangle = \begin{cases} 2\pi m \left(\frac{\partial \langle \hat{\mathcal{C}}(\boldsymbol{x}, t)\rangle}{\partial \ln a(0,0)}\right)_s & (2\mathrm{D}), \\ -4\pi m \left(\frac{\partial \langle \hat{\mathcal{C}}(\boldsymbol{x}, t)\rangle}{\partial a^{-1}(0,0)}\right)_s & (3\mathrm{D}), \end{cases} \quad (6)$$

at constant entropy per particle $s = S/N$. The time dependent contact response captures how quickly the contact adjusts to a change in scattering length; this directly determines the dynamical bulk viscosity according to Eq. (5). This makes the contact correlation, and hence the dynamical bulk viscosity, directly accessible in cold atom experiments where the scattering length can be controlled in time by the magnetic field near a Feshbach resonance and the time evolution of the contact has already been measured using rf spectroscopy [38,39].

Viscosity sum rule.—Since the pressure operator is Hermitian, the dynamical bulk viscosity is an even and positive function of frequency, $\zeta(\omega) \geq 0$ [8]. The integral over all frequencies in Eqs. (5), (6) immediately yields the bulk viscosity sum rule [8,10] with $\mathcal{C} = \langle \hat{\mathcal{C}} \rangle$,

$$S \equiv \frac{2}{\pi} \int_0^\infty d\omega \zeta(\omega) = \begin{cases} -\frac{1}{8\pi m} \left(\frac{\partial \mathcal{C}}{\partial \ln a}\right)_s & (2\mathrm{D}), \\ \frac{1}{36\pi m a^2} \left(\frac{\partial \mathcal{C}}{\partial a^{-1}}\right)_s & (3\mathrm{D}). \end{cases} \quad (7)$$

Using the Tan adiabatic relation to express the contact $\Pi_{\mathrm{an}} = c_d \mathcal{C} = (\partial \mathcal{E}/\partial \ln|a|)_s$ as the scale variation of the energy density $\mathcal{E}$ [17,37,40], the sum rule is given by the scale "susceptibility" $S = -(1/d^2)[\partial^2 \mathcal{E}/\partial(\ln|a|)^2]_s \geq 0$ in d dimensions. The sum rule is taken at constant entropy per particle to ensure that the bulk viscosity of the ideal gas is zero [8].

Pair fluctuations.—The local contact density can equivalently be interpreted as the density operator $\hat{\mathcal{C}} = \hat{\Delta}^\dagger \hat{\Delta}(\boldsymbol{x})$ of the local fermion pair field $\hat{\Delta}(\boldsymbol{x}) = m g_0 \psi_\downarrow(\boldsymbol{x}) \psi_\uparrow(\boldsymbol{x})$. The bulk viscosity thus depends directly, and only, on pairing fluctuations within the attractive Fermi gas; it is given exactly by the four-point pair correlation function $\chi(\boldsymbol{x}, t) = \langle [\hat{\Delta}^\dagger \hat{\Delta}(\boldsymbol{x}, t), \hat{\Delta}^\dagger \hat{\Delta}(0,0)] \rangle$. One can anticipate that the bulk viscosity has a strong signature at the superfluid phase transition which is driven by pair fluctuations (see below). While pair fluctuations are strong also at unitarity, the prefactor $c_d^2 \sim 1/a^2$ ensures that ζ vanishes in this case.

To summarize, the bulk viscosity is the response function of the trace anomaly and is therefore sensitive to scaling violation. For the dilute quantum gas, the trace anomaly is proportional to the contact density of local pairs and depends only on the pairing properties. This establishes the link between pairing [41] and the quantum scale

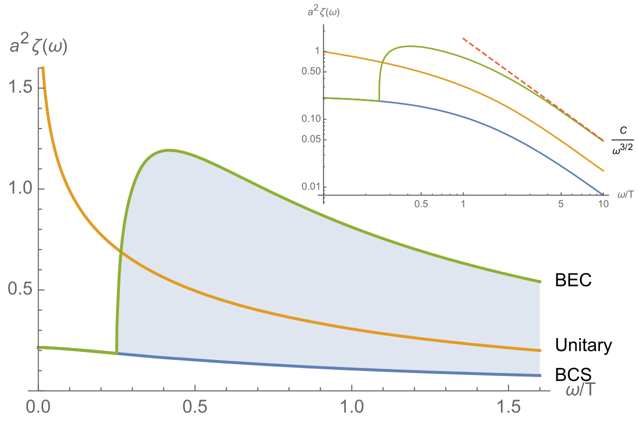

FIG. 1. Dynamical bulk viscosity $\zeta(\omega)/[\sqrt{2}z^2/9\pi a^2\lambda]$ vs frequency ω in the high-temperature limit (11). From top to bottom: BEC ($v = 0.5$, green), unitary ($v = 0$, orange), BCS ($v = -0.5$, blue). Inset: logarithmic plot shows exact high-frequency asymptotics (10) proportional to $\mathcal{C}/\omega^{3/2}$ (dashed).

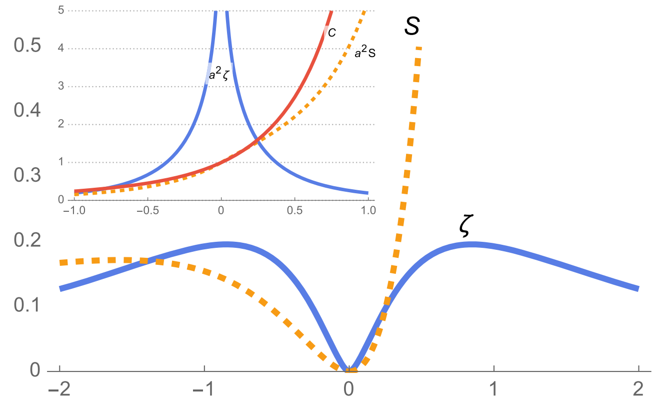

FIG. 2. Bulk viscosity $\zeta(v)$ vs interaction $v = \lambda/a\sqrt{2\pi}$ in the high-temperature limit. Viscosity $\zeta/[2^{3/2}z^2/9\pi\lambda^3]$ (14) (solid blue), sum rule $S/[2^{3/2}z^2T/9\lambda^3]$ (12) (dashed orange). Inset: Contact correlation $a^2\zeta$ (solid blue), contact sum rule a^2S (dashed orange), and contact $\mathcal{C}/[16\pi z^2\lambda^{-4}]$ (red).

anomaly [21] suggested by recent experiments in 2D Fermi gases.

Analytical results.—The contact correlations and bulk viscosity can be computed exactly in several limiting cases: at (i) zero density (two-body), (ii) high frequency, and (iii) high temperature (virial expansion).

The zero-density case (i) is determined solely by two-body physics. In this limit, the only source of dissipation is the dissociation of a bound molecule at the two-body binding energy $\varepsilon_B = \hbar^2/ma^2$; this yields a high-frequency tail above the threshold $\omega > \varepsilon_B$ to break a pair [10,42],

$$\zeta_{\text{2D,vac}}(\omega) = \frac{\mathcal{C}_0}{4m\omega} \times \frac{\Theta(\omega - \varepsilon_B)}{\ln^2(\omega/\varepsilon_B - 1) + \pi^2}, \tag{8}$$

in 2D, where $\mathcal{C}_0$ denotes the two-body contact. In 3D, a two-body bound state exists only on the BEC side for $a > 0$, and

$$\zeta_{\text{3D,vac}}(\omega) = \frac{\mathcal{C}_0\Theta(a)}{36\pi ma}\frac{\sqrt{\varepsilon_B(\omega - \varepsilon_B)}\Theta(\omega - \varepsilon_B)}{\omega^2}. \tag{9}$$

This two-body result serves to disentangle the dissipation due to two-body pair breaking from the genuine many-body bulk viscosity below [10].

In the limit (ii) of high frequency $\omega \gg \varepsilon_F, T$, the contact correlator is evaluated at small times where it factorizes as $\chi(\boldsymbol{x}, t \to 0) \simeq m^2\Gamma(\boldsymbol{x}, t)\mathcal{C}(0,0)$; at large frequency, the pair propagator $\Gamma(\boldsymbol{x}, t)$ approaches the zero-density form [42]. It follows immediately that for $\omega \to \infty$ the bulk viscosity is proportional to the contact density and decays with a characteristic frequency dependence,

$$\zeta(\omega \to \infty) = \begin{cases} \frac{\mathcal{C}}{4m\omega\ln^2(\omega/\varepsilon_B)} & \text{(2D)}, \\ \frac{\mathcal{C}}{36\pi a^2(m\omega)^{3/2}} & \text{(3D)}. \end{cases} \tag{10}$$

This derivation reproduces earlier results [10,43,44] in a dramatically simpler calculation. The zero-density results (8) and (9) approach the high-frequency limit with two-body contact density $\mathcal{C}_0$. However, the exact high-frequency limit is more general and holds at arbitrary density, temperature and interaction in terms of the total contact density $\mathcal{C}(n, T, a)$. This asymptotic behavior is important because it guarantees convergence of the sum rule (7).

Finally, the dynamical bulk viscosity can be computed exactly in the high-temperature limit (iii) by virial expansion [11,12]. To second order in fugacity $z = e^{\beta\mu}$, the pair distribution $b(\varepsilon) = z^2e^{-\beta(\varepsilon+2\mu)}$ is combined with the zero-density spectral function to yield [42]

$$\begin{aligned}\zeta_{\text{3D,vir}}(\omega > 0) = {}& \frac{2\sqrt{2}}{9}z^2\lambda^{-3}v^2\frac{1 - e^{-\beta\omega}}{\beta\omega} \\ & \times \left(\Theta(v)2ve^{v^2}\frac{\sqrt{\beta\omega - v^2}\Theta(\beta\omega - v^2)}{\beta\omega}\right. \\ & \left. + \frac{1}{\pi}\int_0^\infty dy\frac{e^{-y}\sqrt{y(y + \beta\omega)}}{(y + v^2)(y + \beta\omega + v^2)}\right).\end{aligned} \tag{11}$$

Here, $v = (\lambda/a)/\sqrt{2\pi}$ denotes the dimensionless interaction parameter as the inverse scattering length in units of the thermal length $\lambda = \sqrt{2\pi/mT}$. The dynamical viscosity has two terms as illustrated in Fig. 1. The first, bound-continuum contribution occurs only on the BEC side $v > 0$ and arises from breaking up bound states at high frequency $|\omega| > \varepsilon_B$, which leads to strong damping as seen

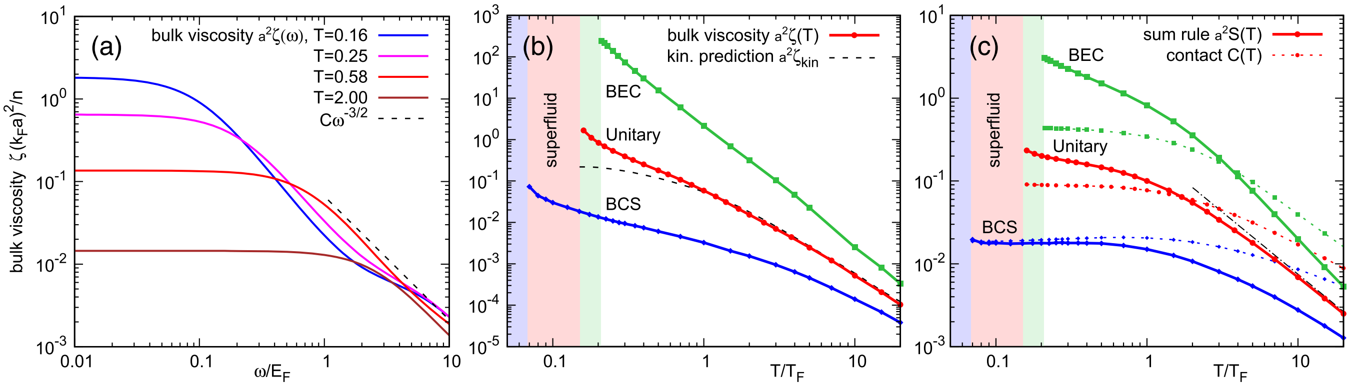

FIG. 3. Bulk viscosity from Luttinger-Ward computation. (a) Dynamical bulk viscosity $\zeta(\omega)(k_F a)^2/\hbar n$ vs frequency ω at unitarity, for increasing temperature from top to bottom (see legend); universal high-frequency tail $\zeta(\omega) \sim \mathcal{C}/\omega^{3/2}$ (dashed). (b) dc bulk viscosity $\zeta(T)(k_F a)^2/\hbar n$ vs temperature for different interaction: from top to bottom BEC ($1/k_F a = 1$), unitary ($1/k_F a = 0$), and BCS ($1/k_F a = -1$). Kinetic prediction $\zeta_{\text{kin}}(T)(k_F a)^2/\hbar n$ (16) from shear viscosity (dashed). (c) Sum rule $S(T)(k_F a)^2/n\varepsilon_F$ (solid) and contact $\mathcal{C}(T)/k_F^4$ (dashed) vs temperature for different interaction; virial limit $T^{-3/2}$ (dot-dash).

before in the two-body limit (9). The second term is the continuum-continuum contribution of dissociated pairs, which extends over all frequencies but has most of its spectral weight at small frequencies $\omega \lesssim \varepsilon_B$. At this order there is no bound-bound contribution because an ideal Bose gas of bound pairs is scale invariant; corrections arise from atom-dimer scattering at order $\mathcal{O}(z^3)$. Both contributions in Eq. (11) are necessary to exhaust the sum rule (cf. Fig. 2)

$$S_{\text{3D,vir}} = \frac{2\sqrt{2}}{9} z^2 T \lambda^{-3} v^2 \left[(1+2v^2) e^{v^2} [1 + \operatorname{erf}(v)] + \frac{2v}{\sqrt{\pi}} - \Theta(v) 4 v^2 e^{v^2} \right]. \tag{12}$$

This agrees with the adiabatic derivative (7) of the contact [11,45] $\mathcal{C}_{\text{3D,vir}} = 16\pi z^2 \lambda^{-4} \{1 + \sqrt{\pi} v e^{v^2} [1 + \operatorname{erf}(v)]\}$.

At unitarity $v \to 0$, the analytical dynamical viscosity

$$\zeta_{\text{3D,vir}}^{\text{unitary}}(\omega) = \frac{2\sqrt{2}}{9\pi} z^2 \lambda^{-3} v^2 \frac{\sinh(\beta\omega/2)}{\beta\omega/2} K_0(\beta\omega/2). \tag{13}$$

At this order, the unitary contact correlation has a logarithmic singularity $a^2\zeta \sim \ln(T/\omega) z^2$ for small frequencies from the modified Bessel function $K_0(\beta\omega/2)$, as shown in Fig. 1. The logarithmic singularity for small frequencies corresponds via Fourier transform to the logarithmic singularity of the bulk viscosity at long times, $\zeta(t) \sim \ln(t)/(a^2 t)$ [46]. Precisely at unitarity, the bulk viscosity vanishes for all frequencies due to the $v^2 \propto a^{-2}$ factor. Throughout the BEC-BCS crossover, the dc bulk viscosity is then given by (see Fig. 2)

$$\zeta_{\text{3D,vir}}(v) = \frac{2\sqrt{2}}{9\pi} z^2 \lambda^{-3} v^2 [-1 - (1+v^2) e^{v^2} \operatorname{Ei}(-v^2)]. \tag{14}$$

The exponential integral $\operatorname{Ei}(x)$ yields a logarithmic singularity in scattering length $a^2\zeta \sim \ln(a^2/\lambda^2) z^2$ shown in the inset of Fig. 2. The singular coefficient of the virial expansion is regularized by higher-order terms $\mathcal{O}(z^3)$ from the fermionic self-energy [9,11]; these are resummed in the Luttinger-Ward computation and yield a finite dc limit in Fig. 3(b) below.

In 2D, there is always a bound state with binding energy $\varepsilon_B > 0$ even for arbitrarily weak attractive interaction. The dynamical bulk viscosity is obtained as [42]

$$\zeta_{\text{2D,vir}}(\omega) = 2\pi z^2 \lambda^{-2} \frac{1 - e^{-\beta\omega}}{\beta\omega} \left[\frac{\beta\varepsilon_B e^{\beta\varepsilon_B} \Theta(\omega - \varepsilon_B)}{\ln^2(\omega/\varepsilon_B - 1) + \pi^2} + \int_0^\infty dy \frac{e^{-y}}{[\ln^2(yT/\varepsilon_B) + \pi^2][\ln^2((yT+\omega)/\varepsilon_B) + \pi^2]} \right].$$

The dc bulk viscosity is then approximately given by

$$\zeta_{\text{2D,vir}}(\varepsilon_B/T) \simeq \frac{2\pi z^2 \lambda^{-2}}{[\ln^2(T/2\varepsilon_B) + \pi^2]^2}. \tag{15}$$

This result for the bulk viscosity based on contact correlations is similar in structure to the fermionic Boltzmann calculation [12] but larger by a factor $4\pi^2$, which is necessary to satisfy the sum rule [42] and the high-frequency asymptotics with the contact density [47,48] $\mathcal{C}_{\text{2D,vir}} = 16\pi^2 z^2 \lambda^{-4} \{\beta\varepsilon_B e^{\beta\varepsilon_B} + \int_0^\infty dy [e^{-y}/(\ln^2(yT/\varepsilon_B) + \pi^2)]$.

Luttinger-Ward results.—The Luttinger-Ward (LW) technique is a diagrammatic strong-coupling approach to fermions in the BEC-BCS crossover [49,50] which treats fermions ψ_σ and the pair field Δ on equal footing. Its predictions for the unitary shear viscosity [9] agree well with recent data [51], and similarly for spin diffusion [52,53]. In this work, I extend the previous LW approach to compute the bulk viscosity (5) via the contact correlation function (6). It uses the self-consistent pair propagator Γ and includes vertex corrections which represent the scattering between pairs, resummed to arbitrary order [42]. While contact vertex corrections are subleading in the high-temperature limit and could be neglected, they are crucial in the quantum degenerate regime and need to be included for an accurate numerical solution.

The *dynamical* bulk viscosity $\zeta(\omega)$ determines the dissipation when the scattering length in Eq. (6) is modulated at frequency ω; the hydrodynamic limit is obtained for $\omega \to 0$. While $\zeta(\omega)$ vanishes at unitarity as $1/a^2$, the contact correlations $a^2\zeta(\omega)$ are nonzero at unitarity as shown in Fig. 3(a). At low temperature there is a pronounced peak at low frequencies $\omega \lesssim T$ that crosses over into the universal high-frequency tail $\zeta(\omega) \sim \mathcal{C}\omega^{-3/2}$ (dashed) for $\omega \gtrsim \varepsilon_F$. At higher $T \gtrsim T_F$ the thermal peak for $\omega \lesssim T$ leads directly into the tail. The peak width $\propto T$ is consistent with quantum critical scaling.

The dc bulk viscosity $a^2\zeta(T)$ shown in Fig. 3(b) is one of the central results: it is largest near the superfluid transition and decreases toward high temperature where pair fluctuations become weaker, as discussed below.

Bulk to shear ratio.—At high temperature kinetic theory predicts the ratio of bulk viscosity ζ to shear viscosity η,

$$\zeta/\eta \propto (\Pi_{\text{an}}/\Pi_{ii})^2 = [(P - 2\mathcal{E}/3)/P]^2, \qquad (16)$$

to be proportional to the squared pressure deviation from scale invariance [11,54]. Using LW bulk, shear and thermodynamic data [9], this is tested by comparing ζ to the kinetic theory prediction $\zeta_{\text{kin}} \equiv \eta[(P - 2\mathcal{E}/3)/P]^2$, which is shown in Fig. 3(b) as the dashed line. There is very good agreement with a proportionality factor of 1 at high temperature $T \geq T_F$, where a quasiparticle picture is expected to hold. Consequently, the shear viscosity at high temperature is fully determined by scale breaking pair fluctuations as reflected in ζ and in the contact.

In the quantum degenerate regime, the bulk viscosity grows monotonically as the temperature is lowered toward the superfluid phase transition and can reach large values $\zeta \gtrsim \hbar n$ near T_c. At low temperature, $\zeta > \zeta_{\text{kin}}$ and also $\zeta/\eta > 1$ can exceed unity since pair fluctuations near the superfluid phase transition affect the bulk viscosity more strongly than the shear viscosity.

Critical pair fluctuations.—The fact that the bulk viscosity is the dynamical correlator of order-parameter fluctuations $\Delta(\boldsymbol{x}, t)$ suggests that ζ might diverge at T_c [29]; instead, vertex corrections in the LW calculation substantially reduce the contact vertex at low momenta and render the bulk viscosity large but finite [42]. The absence of divergent critical scaling might depend on how the critical point is approached, as found in QCD [55].

Finally, Fig. 3(c) shows the viscosity sum rule S. It is large in the quantum degenerate regime and decreases toward high temperature as $T^{-3/2}$ (12) (dot-dashed), i.e., faster than the contact $\mathcal{C} \sim T^{-1}$ itself (dashed) [9,45,56].

To conclude, the bulk viscosity identifies the breaking of scale invariance with the strength of pair fluctuations, which become very large near T_c and on the BEC side. This provides a strong signature in cold atom experiments, either directly in the response of the contact [14,38,39] to a change in scattering length, or by modulating the scattering length periodically and measuring the dissipative heating rate $\dot{\mathcal{E}} = d^2(\partial_t a^{-1})^2 a^2 \zeta$ [14] proportional to $a^2\zeta(\omega, T)$ shown in Figs. 3(a),3(b), which is nonzero also at unitarity. Further signatures of enhanced dissipation ζ can be found in the hydrodynamic description of scaling or breathing dynamics [10,13,18,21] and sound attenuation $D_s = [\frac{4}{3}\eta + \zeta + \kappa(c_v^{-1} - c_p^{-1})]/mn$ [2,57].

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Note added.—After submission, two other calculations [58,59] of the bulk viscosity in the high-temperature limit appeared, which agree with our results where applicable.

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