

# Tuning and Backreaction in $F$ -term Axion Monodromy Inflation

( A. Hebecker, Heidelberg)

in collab. with **P. Mangat F. Rompineve and L. Witkowski**

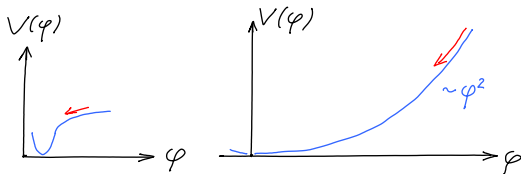
## Outline

- Why look for large-field inflation models?
- Proposals: KNP vs. axion-monodromy models
- Recent progress:  $F$ -term axion monodromy
- Tuning requirements and their implementation in  $F$ -theory
- Backreaction; Kahler moduli stabilization; landscape restrictions

## Preliminaries

- Slow-roll inflation comes in two variants:  
**small-** and **large-field** models

(Always in units where  $\overline{M}_P = 1$ )



- Small-field models require a (tuned) very flat potential
- Large-field models work with generic potentials (e.g.  $V(\varphi) \sim \varphi^2$ ), but the field-range  $\Delta\varphi \gg 1$  is a challenge

## 'Why look for large-field models in string theory?'

### 1) Observations

- The amount of primordial gravity waves is measured by the tensor-to-scalar ratio:

$$r = \frac{\Delta_T^2}{\Delta_R^2} \simeq 8 \left| \frac{d\varphi}{dN} \right|^2 \quad \Rightarrow \quad \Delta\varphi \simeq 20\sqrt{r}$$

- Thus, even though the BICEP 'discovery' went away, the need to consider large-field models may return
- Note: The new Planck/BICEP analysis still sees a ( $\sim 1.8\sigma$ ) hint for  $r \simeq 0.05$
- Much better values/bounds are expected soon

## 'Why look for large-field models in string theory?'

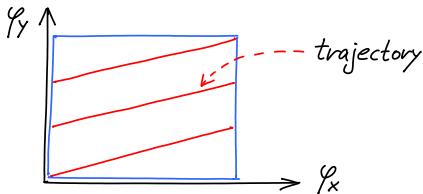
### 2) Fundamental

- Do (parametrically) large-field models exist in consistent quantum gravity theories?  
see e.g. Arkani-Hamed/Motl/Nicolis/Vafa '06 .... Conlon '12
- Do they exist in the type IIB / F-theory landscape as we understand it at present?
- Basic obstacle: Moduli spaces of string compactifications are 'essentially' compact  
  
(Note: Of course, specific non-compact directions exist, e.g. large-volume or large-complex-structure. However, in these directions the potential decays way too quickly.)

## Kim-Nilles-Peloso mechanism

Kim, Nilles, Peloso '04

- One such idea is to realize a 'winding' trajectory on a 2d periodic field space:

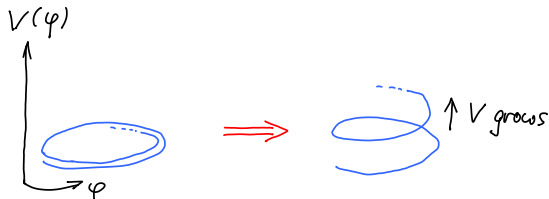


- Clearly, such a trajectory can be much longer than the (naive) field range
- The technical challenge is the realization of the required potential in concrete string models

Our focus here: Monodromy inflation

Silverstein/Westphal/McAllister '08

- We start with a single, periodic inflaton  $\varphi$
- The periodicity is then **weakly** broken by the scalar potential



- Various concrete stringy realizations have been discussed; for an F-theoretic suggestion see

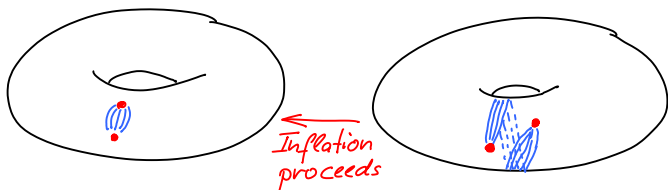
Palti/Weigand '14

## F-term axion monodromy

- Very recently, the first suggestions have emerged how this could be realized in a quantitatively controlled way (i.e. in a 4d supergravity description, stabilized moduli)

Marchesano/Shiu/Uranga '14  
Blumenhagen/Plauschinn '14  
AH/Kraus/Witkowski '14

- In particular, in **our** suggestion inflation corresponds to **D7-brane-motion**
- The monodromy arises from a flux sourced by the brane



## F-term axion monodromy (continued)

- One starts with shift-symmetric Kahler potential

$$K = K(u - \bar{u})$$

- Concretely, this can be realized in the large-complex structure limit of a 3-fold or 4-fold (where  $u$  could be a brane position)

Arends, AH, Heimpel, Kraus, Lüst, Mayrhofer, Schick, Weigand  
McAllister, Silverstein, Westphal, Wrase  
Blumenhagen, Herrschmann, Plauschinn  
Hayashi, Matsuda, Watari '14

see Garca-Etxebarria, Grimm, Valenzuela for possible alternatives

- The shift symmetry is broken (and a monodromy introduced) by e.g. a flux choice

$$W = w + au,$$

- To keep this effect small, one needs small  $a$



## F-term axion monodromy (continued)

- Complex structure moduli  $\{z^i\}$  other than  $u$  need to be included:

$$W = w(z) + au$$

- They can be much heavier than  $\text{Re}(u)$ , if  $a \sim 1$  and  $w \gg 1$

Blumenhagen, Herrschmann, Plauschinn '14

- However, the inflationary energy is then still high and Kahler moduli stabilization is problematic
- Thus, we want  $a \ll 1$ , which requires  $a = a(z)$

AH, Mangat, Rompineve, Witkowski '14

## Tuning in $F$ -term axion monodromy

- Thus, we must consider the structure

$$K = K(z, \bar{z}, u - \bar{u}) \quad , \quad W = w(z) + a(z)u \quad ,$$

with  $a(z) \ll 1$  at the starting point  $DW = 0$

- We appeal to the standard no-scale cancellation in the Kahler moduli sector
- The scalar potential is then determined by the (two)  $F$ -terms

$$D_u W = D_u w + a + K_u a u$$

$$D_z W = D_z w + (\partial_z a + K_z a) u$$

- With  $y \equiv \text{Re}(u)$ , we find

$$V \sim |K_u a|^2 y^2 + |\partial_z a + K_z a|^2 y^2 + \dots$$

## Tuning in $F$ -term axion monodromy (continued)

- To keep the potential

$$V \sim |K_u a|^2 y^2 + |\partial_z a + K_z a| y^2 + \dots$$

flat, we need to tune **both**  $a \ll 1$  and  $a_z \ll 1$

- Depending on how many of the  $z^i$  actually enter  $a(z)$ , the tuning price can be very high
- We believe that this is a generic feature of  $F$ -term axion monodromy models (although our explicit analysis is limited to complex-structure and F-theory/D7-brane models)

## Towards concrete realizations

- Let us write  $\{z^i, u\} \equiv \{z^I\}$  and consider the 3-fold period vector

$$\Pi_\alpha = \begin{pmatrix} 1 \\ z^I \\ \frac{1}{2}\kappa_{IJK}z^Jz^K + \sum_p A_{Ip}e^{-\sum_J a_{pJ}z^J} \\ -\frac{1}{3!}\kappa_{IJK}z^Iz^Jz^K + \sum_p B_{pI}e^{-\sum_J b_{pJ}z^J} \end{pmatrix}$$

as well as Kahler and superpotential

$$K = -\ln(S - \bar{S}) - \ln \left[ \Pi_\alpha(z, u) \bar{\Pi}^\alpha(\bar{z}, \bar{u}) \right]$$

$$W = (N_F - SN_H)^\alpha \Pi_\alpha(z, u)$$

## Towards concrete realizations (continued)

- Assuming that (at least)  $u$  is in the large-complex-structure limit,  $W$  takes the form

$$W = w(S, z) + a(S, z)u + \frac{1}{2}b(S, z)u^2 + \frac{1}{3!}c(S)u^3$$

- Moreover,

$$c(S) \sim (m + nS),$$

with  $m, n \in \mathbb{Z}$  and  $S = \frac{i}{g_s} + C_0$ .

Thus, a tuning  $c \ll 1$  is impossible and  $c$  must be set to zero.

- $b(S, z)$  has a piece linear in  $z$ , with an  $S$ -dependent prefactor. A similar argument can again be made.
- This goes on and the whole structure collapses to  $a = b = c = 0$ .

## Towards concrete realizations (continued)

- The above was oversimplified. The actual no-go theorem for tuning the coefficients of  $u^n$  in  $W$  relies on
  - a) The maximally cubic field dependence (at LCS)
  - b) The linear additional  $S$ -dependence
- These conditions are violated in F-theory fourfold at LCS
- The period vector is structurally as above, just with 4-th order polynomials
- As a result, for fourfolds it is in principle possible to realize

$$K = K(z, \bar{z}, u - \bar{u}) \quad , \quad W = w(z) + a(z)u \quad ,$$

with  $a(z) \ll 1$  and  $a_z(z) \ll 1$  in a SUSY vacuum

## Backreaction

- The scalar potential

$$V = e^K \left( K^{I,J} D_I W \overline{D_J W} \right)$$

can be worked out and, with  $u = ix + y$ , takes the schematic form

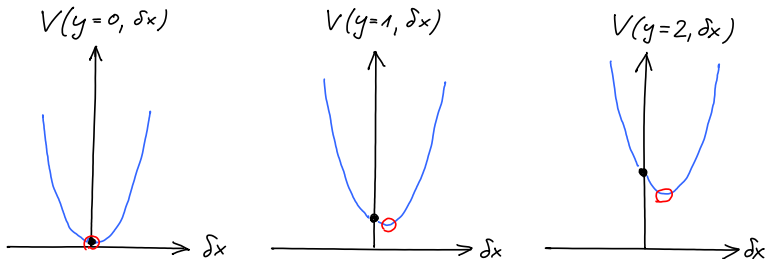
$$V = A(z, \bar{z}, x) + B(z, \bar{z}, x) y + C(z, \bar{z}, x) y^2$$

(in the SUSY vacuum  $\{z_0, \bar{z}_0, x_0, y_0 = 0\}$  we have  $A = B = 0$ )

- At large  $y$ , certain field displacements  $\delta z \equiv z - z_0$  etc. arise
- Since the 'naive' potential is very flat ( $C(z_0, \bar{z}_0, x_*) \ll 1$  by tuning), even small  $\delta z$  induce  $\mathcal{O}(1)$  corrections.

## Backreaction (continued)

- The backreacted potential arises by minimizing  $V(z, \bar{z}, x, y)$  with respect to  $\{z, \bar{z}, x\}$  at each  $y$ :



- Thus, while the 'naive' potential is by definition quadratic, the backreacted potential is **flatter** and could potentially even become **non-monotonic**

cf. related considerations by Dong, Horn, Silverstein, Westphal, '10



## Backreaction (continued)

- Specifically, we tune  $a \sim \epsilon$  and  $(a_z + K_z a) \sim \epsilon^2$  (with  $\epsilon \ll 1$ )
- Working up to quadratic order in  $\delta z^i$  (and writing  $z^i = v^i + iw^i$ ) we find

$$V = \frac{1}{2} \Delta^T \mathcal{D}(y) \Delta + b^T(y) \Delta + \mu^2 y^2 ,$$

where

$$\Delta = \{\delta x, \delta v^i, \delta w^i\}$$

and  $\mathcal{D}$ ,  $b$  are complicated (but in principle explicit) matrix and vector-valued expressions

## Backreaction (continued)

- Minimization in  $\{z, \bar{z}, x\}$  gives the fully backreacted potential  $V_{\text{eff}}(y)$  – a very complicated function
- However, at  $1 \ll y \ll 1/\epsilon$ , things simplify and we find

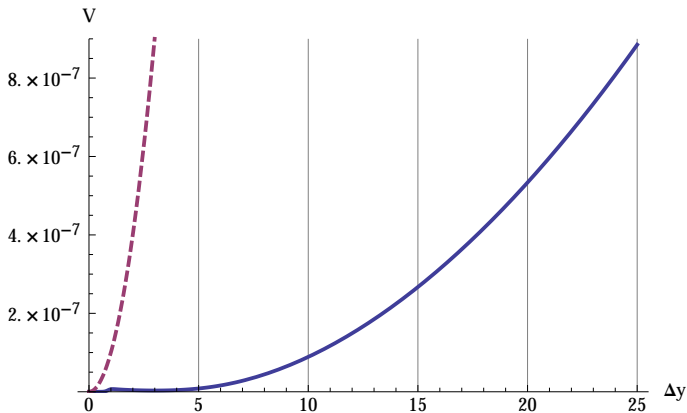
$$V_{\text{eff}}(y) \sim (-\mathcal{O}(1)\epsilon^4 + \mu^2) y^2$$

where also  $\mu^2 \sim \epsilon^4$

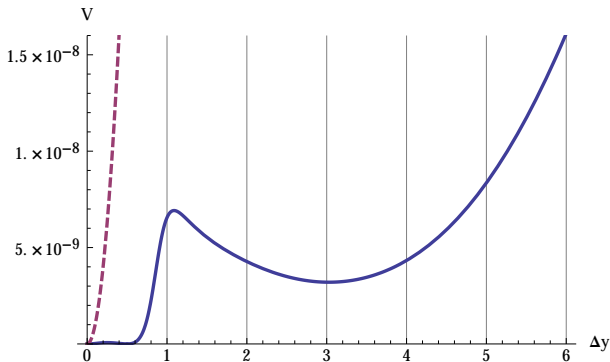
- In this regime, **large-field quadratic** inflation can proceed
- Kahler moduli stabilization à la **LVS** is not disrupted thanks to  $\epsilon \ll 1$

## Illustration of naive and fully backreacted potentials

in a (supergravity level) numerical example



## Blowup of non-monotonic region at (relatively) small $y$



- Note that this might, independently of inflation, be of interest as an  $F$ -term uplifting proposal

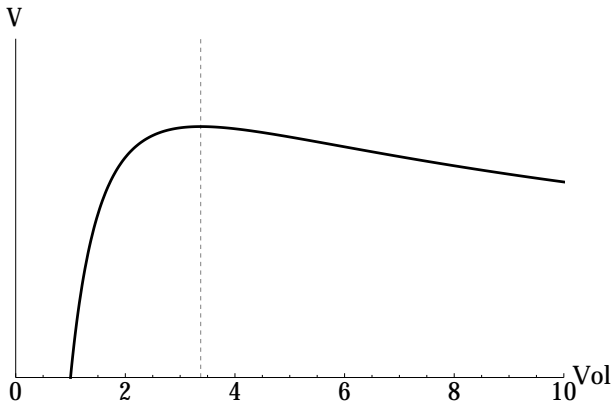
cf. e.g. Kallosh, Linde, Vercocke, Wrase '14

## An alternative, less severe tuning option (very briefly...)

- Replace  $a \sim \epsilon$ ,  $(a_z + K_z a) \sim \epsilon^2$  by  $a \sim a_z \sim \epsilon$
- **As before:** Include backreaction order by order in  $\epsilon$  by determining the shifts  $\{\delta z, \delta \bar{z}, \delta x\}$  at each  $y$
- However, now we find a simple (quadratic)  $y$ -dependence only at  $y \gg 1/\epsilon$
- Hence  $W \sim w + ay$  changes significantly during inflation
- Since, in the LVS, the volume is 'fixed' at  $\mathcal{V} \sim W$ , the universal Kahler modulus now also backreacts
- Nevertheless, **large-field inflation remains possible** (but no pheno yet...)

## An alternative, less severe tuning option – illustration

- Scalar potential with complex-structure and Kahler modulus backreaction (plotted as a function of the volume)



## Can the tuning be realized?

- Is the underlying flux landscape large enough?
- We follow the classical analysis of Denef/Douglas '04
- For a fourfold with a D3-tadpole bound  $L_* = \chi/24$ , they find

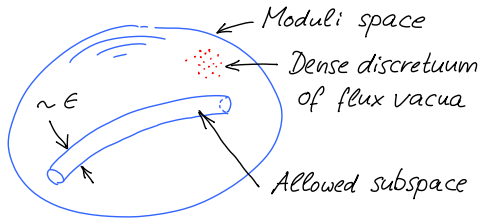
$$\mathcal{N} \sim \frac{(2\pi L_*)^{2m}}{(2m)!} \int_{\mathcal{M}} d^{2m}z \det g \rho(z)$$

SUSY flux vacua

- Here the second factor is an appropriately weighted integral over moduli space (expected to give an  $\mathcal{O}(1)$  number)

## Can the tuning be realized? – Illustration

We need to be inside a tubular region (of size  $\epsilon$ ) around the submanifold defined by  $a = a_{z^i} = 0$





## Can the tuning be realized? (continued)

- In our application,  $\mathcal{N}$  is replaced by

$$\mathcal{N}(|a_I| < \epsilon) \sim \frac{(2\pi L_*)^{(b_4 - J_f - J_t)/2}}{(b_4 - J_f - J_t)/2!} \cdot (\pi\epsilon^2)^{J_t} \times \mathcal{O}(1)$$

where  $b_4$  defines the dimension of the (4-form) flux space,

$J_f$  subtracts the number of fluxes forbidden by the F-theory limit and the assumed linearity of  $W$  as a function of  $u$

$J_t$  counts the tuning conditions (i.e. how many of the moduli appear in  $a(z)$ )

- For e.g.  $\varphi_{max} \sim 15$ ,  $L_* \sim 900$ ,  $b_4 \sim 23000$ , and assuming that 300 of the 3800 moduli appear in  $a$ , we find that  $10^{300}$  of the  $10^{1700}$  flux vacua survive

see Denef '08 for this example

## Summary/Conclusions

- Large-field inflation is a challenge and an opportunity for string theory
- This remains true even if the tensor modes (or field-range) are way below last year's BICEP claim
- In 'our' variant of F-term axion monodromy, a high tuning price has to be paid (and we don't know of an equally 'complete' and less tuned version)
- We need the  $\kappa_{IJKL}$ 's of a proper F-theory 4-fold (and, ideally, also the subleading (non-instantonic) terms in the periods)
- Need a better 10d/stringy understanding  
developing e.g. recent work of Ibanez, Marchesano, Valenzuela