Fundamentals of Orbifold GUTs

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based partially on work in collab. with

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Outline

1. Introduction

   Motivation; basic principles; simple models

2. Group Theory

   Breaking by inner automorphism; rank reduction, matter from gauge

3. Geometry

   5- and 6d geometries, generalizations of orbifold framework, non-local breaking

Motivation

On the one hand:

- extra dimensions – rich framework for building highly symmetric, potentially realistic models
- powerful geometric symm. breaking mechanisms (Hosotani, Scherk-Schwarz, brane-to-brane mediation)
- arise in string theory; can also be discussed consistently in eff. field theory

On the other hand:

- low-energy SUSY remains most successful approach to hierarchy problem
- in minimal model, SM gauge couplings meet at $\sim 10^{16}$ GeV
- interpreting ν-masses within the SM (as an eff. field theory), the operator $\frac{1}{M}(HL)^2$ points to $M \sim 10^{15}$ GeV

$\Longrightarrow$ strong motivation for 4d eff. field theory description up to $10^{15 \cdots 16}$ GeV + higher dim.s at that scale (i.e., orbifold GUTs)

Basic structure

(Compactif. by "modding out" discrete symmetry)

$\mathbb{Z}$-transl. :

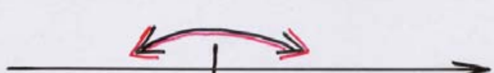
$\Rightarrow S^1 = \mathbb{R}/\mathbb{Z}$

$\mathbb{Z} \to$ R-symm. — Scherk-Schwarz-Breaking

$\mathbb{Z} \to G$ (gauge gr.) — Hosotani mechanism

(cf. also CY-comp. of heterotic string)

$\mathbb{Z}_2$-reflection :



$\Rightarrow$ half-line $= \mathbb{R}/\mathbb{Z}_2$; interval $= S^1/\mathbb{Z}_2$

fixed points = singularities = points with reduced symmetry

in string theory: singularities do not destroy UV-completeness of the theory

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in field theory: more freedom in model building

but: • some loss of predictive power

• more UV sensitivity

SU_5 in 5d

Kawamura '00 ...

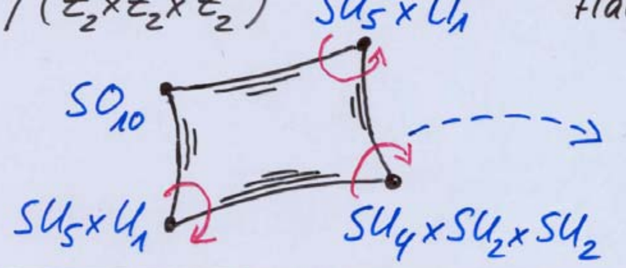
$S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$: SU_5 |——SU_5——| SM

$\mathbb{Z}_2'$ linked to $P = \mathrm{diag}(1,1,1,-1,-1) \in SU_5$

SO_{10} in 6d

Asaka, Buchmüller, Covi; Hall, Nomura, Okui, Smith '01

$T_2/(\mathbb{Z}_2 \times \mathbb{Z}_2' \times \mathbb{Z}_2'')$



→ Wilson lines with values $P_i \in SO_{10}$

- light Higgs doublet (triplet naturally heavy!) from bulk 5 (cf. Witten '85; ...)
- or simply light doublet from brane (no triplet exists!) (A.H., March-Russell '01)
- dim-5 proton decay absent (could even be completely forbidden) (Altarelli, Feruglio; Hall, Nomura '01)
- unification at $\sim 10^{17}$ GeV (with $\frac{1}{R} \sim 10^{15}$ GeV)

  unless: power-law effects

  (Dienes, Dudas, Ghergetta '98 ... A.H., Westphal '02)

Group Theory - Fundamentals

spatial trf. $k: x \to x'$ $\longleftrightarrow$ gauge twist $P \in G$

$$A_\mu(x) = P A_\mu(k^{-1}x) P^{-1}$$

$\Rightarrow$ surviving group H defined by $h = P h P^{-1}$

one twist: $P = e^T$; $T \in$ Cartan subalg. (G)

$\Rightarrow$ rank preserved

two (or more) twists: $P = e^T$; $P' = e^{T'}$

for $[T, T'] \neq 0$, rank reduction will in general occur

interesting option: $[T, T'] \neq 0$

yet $P.P' = P'P$

$\Rightarrow$ rank reduction on abelian orbifolds is possible

(more details in talk of M. Ratz)

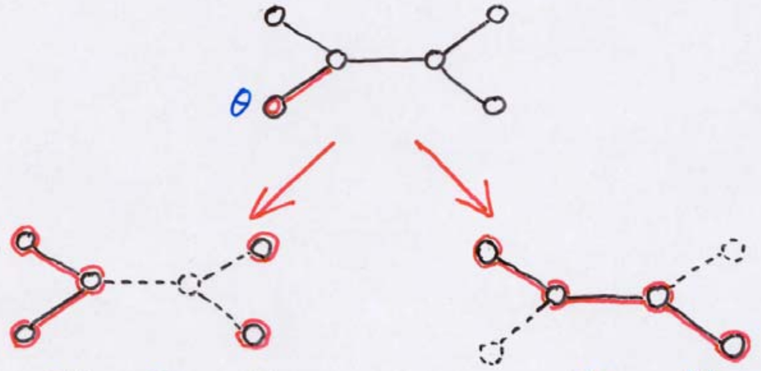
further: one-step rank reduction + other possibilities in outer automorphisms for systematic study:

Dynkin diagram techniques

simple roots +
most negative root θ

Katsuki et al. '89
Choi, Hwang, Kim, '03
A.H., Ratz, '03

for example: SO_{10}



$SU_4 \times SU_2 \times SU_2$ — twist P_1

$SU_5 \times U_1$ — twist P_2

$\Rightarrow$ the twist $P_1 \cdot P_2$ leads to $SU_5' \times U_1'$

systematic generalization:

$E_6 \longrightarrow SO_{10} \times U_1$; $SU_6 \times SU_2$; $SO_{10}' \times U_1'$

$E_7 \longrightarrow E_6 \times U_1$; $SO_{12} \times SU_2$; $E_6' \times U_1'$

$E_8 \longrightarrow E_7 \times SU_2$; $E_7' \times SU_2'$; $E_7'' \times SU_2''$

Matter and Higgs from gauge

- use extra-dim. components of gauge fields
- also: Watari, Yanagida '02
  Babu, Barr, Kyae, '02
  Burdman, Nomura, '02
  ...

to be more predictive than simplest orbifold GUTs:

- understand branes
- or: derive matter & Higgs from bulk
  (SUSY partners of broken gauge fields)

  ideally: Yukawa couplings from gauge couplings

achieved so far:

6d $N=2$ SYM ; groups E_7, E_8

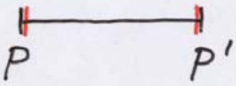
↓

4d $N=1$ models with just 3 generations + Higgs doublets (+ U_1's or flavour SU_3)

(some of these models use generalized methods → conifold GUTs (next!))

Geometry

5d: S^1 or Interval

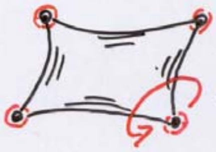


6d: (assuming local flatness)

Torus

Cylinder etc.

Pillow etc.

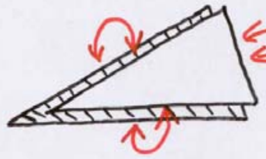


← breaking linked to Wilson lines

(but still fixed by boundary conditions)

more general:

"Conifold GUTs"



(glueing together two arbitrary identical polygons)

⇒ manifold with S^2-topology + conical singularities with arbitr. def. angle + Wilson lines

Non-local breaking

Orbifolds: → points with reduced symmetry

⇒ arbitraryness in field theory

Hosotani mech.: → breaking is non-local

⇒ Wilson-lines are moduli

This is avoided if π_1 is finite!

(familiar in string theory: Dixon et al. '85
Wen, Witten '85)

simplest example: projective plane

sphere with cross cap

cf. Hall, Murayama, Nomura, '01

product of two such loops is contractible

⇒ gauge twist P satisfies $P^2 = 1$

⇒ no Wilson-line modulus

Problem: bulk curvature

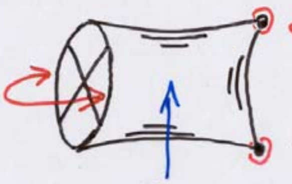
(→ ~~SUSY~~ ; no massless fermions)

possible solution: (in preparation)

Concentrate curvature at conical singularities with full GUT symmetry!

$P = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 & \\ & & & & -1 \end{pmatrix}$

breaks to SM



- global construction as $T_2 / (Z_2 \times Z_2')$
- 4d eff. theory: $SU_3 \times SU_2 \times U_1$; $N=1$ SUSY

remaining problem: doublet-triplet-splitting

solution:
- start with SU_6

(→ Hall, Nomura, Smith ; Paccetti, Schmidt, Tavartkiladze)

- break to $SU_5 \times U_1$ at fixed point
- break to $SU_4 \times SU_2 \times U_1$ ($SU_5 \to SM$) at crosscap (non-locally!)

⇒ $SM \times U_1$ with light Higgs doublets ("from gauge")

Conclusions

Orbifold GUTs – arguably the simplest, fully fully realistic GUT models (5d SU_5, 6d SO_{10})

Price to pay:
- only 1-loop precision in gauge unification
- no generic predictivity in matter/Yukawa sector

Potential:
- improving predictivity (understanding branes, non-local breaking)
- deriving the matter sector (matter/Higgs from gauge, larger groups, more dimensions)

Promising bottom-up approach; complementary to string-based efforts